E-learning in mathematics

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Abstract—**The problems in the e-learning course market (for example their low sales compared to mobile applications) show that besides the development of new technologies, it is necessary to also pay attention to other factors that affect the creation of meaningful courses. These factors are the content of said courses, as well as their real applications as related to their didactic and educational aims. Furthermore, attention has to be paid to the administration and maintenance of courses and the training of educators in their proper use. In this article we aim to show the risks of over-dependence on ICT technologies in the teaching of mathematics, and suggest who should be preparing meaningful e-learning courses.**

I. Introduction

Fifteen years ago it was hard to imagine all that would be achievable using ICT technologies in the year 2011. The accessibility of computers and the internet has increased exponentially. There were some who owned computers, or mobile phones, and some teachers used ICT technologies. Today there are few in Slovakia who do not own a mobile phone or computer, and yet we know no teachers, parents, or students who would purchase an e-learning math course with their own money.

Herein lies the essential problem of the development of technical support for teaching: e-learning courses are created by those who want to sell them, and not those who want to use them to learn. The incorrect motivation then leads to the creation of mathematics courses that mirror the ideas of software developers. To the layman the course appears as mathematics, however it does not lead to the development of the logical thinking of children or the acquiring of other math skills such as the ability to make arguments, propose an experiment, make analogies, systematize knowledge, think critically about conclusions, pose the right questions, and create relationships and connections between knowledge. Instead, mathematics is presented in a “rounded-out”, finite form and structure. When building concepts and knowledge, the progression and the structure of their development are significantly different. This fact is ignored. Mathematics is “demonstrated”, presented and developed through external symbols: elegant presentation, schooled actor voices (who know nothing of what they speak of, and intonate in discord with the logic of the content) etc. Some statements are often even erroneous and incorrect, to the point where it seems that we are presented merely with the viewpoint of the authors of the software, who understand mathematics only formally and superficially.

In this article we will attempt to explore why e-learning lessons often do not support, and sometimes even hinder the teaching of mathematics. We will introduce certain mechanisms of the teaching and learning of this scientific discipline, and the risks that come with ignoring them. We will also propose some principles that could help in creating useful e-learning aids for teaching mathematics. We base these on a constructivist approach towards teaching mathematics. From the rich literature describing and illustrating the approach, we name for example publications [1], [2], [3]. This approach is certainly more difficult and complicated to integrate into e-learning applications than its opposite, called also the transmissive approach. There the chief task of the educator, and the education itself, is the direct transmission of the required, defined area and structure of knowledge “from the mind of the teacher into the mind of the pupil”. The reality that this approach is much more acceptable, even optimal, for e-learning is a further possible main problem.

II. Multimedia Perfection And The Creation of Mathematical Knowledge

Mathematical growth and development occurs, even in the case of reaction to regular schooling, firstly in the mind of the student. The optimal method is the creation and building of concepts on the basis of the child’s own activities, experiences, and discoveries. It should be, in a realistic magnitude, experienced by every elementary school student. Mathematically gifted and talented children of this age are in our experience able to, under expert guidance, independently build and create the majority of the required mathematical knowledge. However, a time commitment beyond usual classroom education is required.

The time requirements therefore lead to the students’ acquiring the majority of material as external, passed on information. However, for quality education it is necessary for the mathematical knowledge to create a so-called cognitive map: it must interconnect both with itself and with the life experiences of the student. Otherwise we build pointless, even harmful, formal knowledge. The newly acquired knowledge needs to be made one’s own, the student must process and confront it with his or her past knowledge. The intensity and quality of this process determines the success of the education.

One of the risks of mathematical e-learning, created by its great multimedia possibilites, is the blocking and suppression of these activities of the student. Psychology gives us a key piece of knowledge: the more senses (and potentially emotions) we engage in order to receive knowledge, the smaller the magnitude of our active mental processing, and conversely the larger the magnitude of our non-critical acceptance, without a corresponding placement within existing structures of experience and knowledge. Imagine, for example, an

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action or romantic scene first as a novel, comic book, black-and-white film and finally as a slick modern movie. Try to compare the methods and results of the mental activity used in processing these inputs.

If our goal is an excellent theatre performance, a powerful film, or an effective commercial, if we want to surprise or shock, then the suppression of the critical thinking and doubting of the viewer is warranted. However, we seriously doubt this is the case in the teaching of mathematics. A polished, multimedia form of presenting and transmitting mathematical knowledge has, based on the described mechanism, and unpleasant but natural tendency to lead to the building of formal, unpersonalized knowledge structures. From the point of view of students who will not significantly utilize mathematics in their future lives, this form of learning is useless. For students who may direct their further studies towards natural sciences and technical fields, it is even harmful. It excludes important learning devices such as

- incorrect formulations and their specification,
- performing unnecessary experiments, and inventing better ones,
- presenting one's own notions and their defense,
- the right to make errors, and the subsequent search for methods by which to reveal and remove said error,
- debating and utilizing analogies.

The listed skills are much more important for doing mathematics, and its utilization in other fields, than repeating definitions and propositions that mean nothing to the students, which they cannot connect to the objects of the real world or to other mathematical concepts.

III. Educational Atmosphere and The Role of the Instructor

. There are two key factors in the quality of mathematical education. The first is the mood, atmosphere of the lessons. This can motivate the students and at the same time harmonize and unify the processes and methods of working of the class. It should be, depending on the circumstances, inspirational, creative, and disciplined. The second factor is the personality of the instructor, his contact and relationship with the students, ability to motivate them, direct their work and discussion.

Much practical experience from classes supported by computers goes against these principles. The teacher devotes a significant amount of energy, time, and attention towards preparing and the subsequent technical execution of the lesson. The actual interaction with students takes the form of mediation, often in the form of prepared methods and presentations deviating from which is often technically not possible or supported. Excessive questions from the students are disruptive and complicate the prepared flow of the lesson. The instructor largely takes on the role of a well trained operator, and much of the opportunity for direct work with the class is lost. The instructor loses much of his ability to observe the overall situation within the class, to create and direct a learning atmosphere, to react directly to the cues and responses of the students. The great effort required to prepare a lesson using ICT technologies leads to an unwillingness to significantly change its contents or direction, even if the situation demands it.

Problems can also arise in the overall mood of the learning process. A classroom equipped with computers is a significantly limiting environment. Working with a PC, an activity not yet quite automated in elementary school students, is distracting, wasting a lot of the attention and energy of both the students and the teacher. Even simply varying levels of competence with a mouse and keyboard can disrupt the progress of the class and decrease attention, especially since the visual attention of the students is focused on the screen. From personal experience we certainly recognize the feeling of concentration and suppression of other stimuli from the moment we place our hands on the keyboard and begin staring at the screen. In this situation it is extremely difficult to impossible for the teacher to adjust the atmosphere and capture the coordinated attention of all the students, to switch between lecturing, discussion, and individual work. Never even mind solving personal problems, cultivating the verbal expression of the students, publicly praising a weaker student for an original solution, directing a bright student who is ridiculing a weaker solution, or praising a student for helping a peer or his courage in contradicting the teacher and pointing out an error.

We have extensive experience with teaching mathematical subjects at a post-secondary level in computer laboratories. We are convinced that teaching mathematics to future computer scientists in the context of their main focus makes sense. However, even for those students it still holds true that the student should only use a computer if it is absolutely necessary. Otherwise, the computers remain off. Additionally, the teacher has an instrument with which to globally control all the screens in the room, or potentially disable the keyboards. Our experience leads us to believe that meaningful work on a PC is possible only when the students have sufficiently understood the essence of the problem and have an idea about their strategies for solving it. This cannot be provided by a PC, to the contrary a PC can be disruptive and a hindrance.

One of the newest innovations is the interactive electronic chalkboard, which has acquitted itself well as a new type of display apparatus on which it is possible to show prepared presentations. It is difficult to use this technology, however, for creative instruction, where it is necessary to react to new ideas and thoughts and often change the expected lesson plan. Classrooms equipped only with electronic chalkboards were, after gaining teaching experience, quickly supplemented with traditional chalkboards.

By contrast, we have had great experiences with primary school students. Here we used ICT applications whose role was to motivate the students towards solving problems that were at the edge of their abilities, that is difficult for their age group. An example of such an experience is described in the article [6]. It is also possible to use ICT to motivate the tiring practice of arithmetic skills, where besides understanding and memorizing the algorithm it is necessary to practice it. This motivation can take various forms, we have used for example games where the student can only make a move after solving a problem.

Similarly, another motivational tool can be an application that actively maintains the student's ranking in the class based on the number and difficulty of problems solved.

In elementary school, the most important thing for the younger students is motivation and feedback from the teacher. His opinions and attitudes are more important to the formation of the students' personality than any formal evaluation. Experience from intensives for mathematically talented children even tells us that the students appreciate authentic behaviour from the instructor such as annoyance, or anger, much mores so than a professional mask of smiles that does not reflect the instructor's true state. How then must the student view instruction at the push of a button, which is exactly the same for every class and every student.

For older students of elementary schools, the tie to society is equally strong, just instead of the teacher it focuses more on the class and their peers.

The aforementioned reasons are why the instructor needs to understand the needs and abilities of the students, to constantly think about their expression and needs, to improve the pedagogic aspects of the instruction, and not least of all for him to be an expert in the subject and theme being taught. The idea that instead of a teacher a lesson could be delivered via a video with an explanation of the subject matter is not only demeaning towards the teacher, but also simply mistaken. A teacher must be truly weak for a filmed lesson to be better than a real one.

These kinds of lectures are however recorded, and do have a justification. For example the [4] provides excellent lectures, which can be used by students who do not have access to traditional education to supplement their knowledge through distance education.

We consider it meaningful to create e-learning courses that an individual can use to set his own tempo of learning, and are focused mainly on developing skills. These can be, for example, first aid courses, typing courses, driving courses. We consider it counterproductive to use mathematics courses consisting of a video recording of a lecture in class. This kind of instruction spoils not only students, but even the teachers.

IV. Evaluating the Effectiveness of E-Learning

T We are interested to know whether there exists, in our context, a generally known and seriously documented case study where e-learning in mathematics classes led to a better result than comparable instruction using other methods. Very often we are faced with arguments that the form and idea are sound, and it is merely the content that needs to be fine-tuned. Possibly other details: students must be conscientious and responsible, they need to be motivated, schools must be willing to invest into technologies. To our generation, this is reminiscent of other unsuccessful attempts to implement an excellent idea, which thankfully since the year 1989 no longer continue in Slovakia.

We want to believe that there are positive examples. However, thus far we have several opposing experiences. In the year 2007 we heard at this conference a contribution concerning a realized e-course of a university level math subject. During the course of discussion we asked whether the success and effectiveness of the course was measured against a concurrently running standard course, which would be easily done using a final exam. Instead of an answer, we received a counter-question whether we mean to doubt the usefulness of ICT in education. Another, very current, experience shows that verifying the suitability and effectiveness of e-learning products more resembles a well planned marketing opportunity. The "testing" is done on a large group of students, teachers, and schools, however during its course they are motivated and rewarded in a way that damages objectivity. The experience and methods of IT firms easily change testing into a process for creating a strong group of fans and lobbyists. Once again, there is a distinct lack of opportunity to compare against a control group using traditional methods. The basis for evaluation should be comparing the results of students with and without using the product in question. The evaluation should be based on an objective measure, not simply the opinions of selected teachers.

We think that real, honest testing of the success of the methods and products of mathematical e-learning is very important, and should also be in the interest of their creators. This is one of the paths to improving the electronic teaching of more than just mathematics. The current absence, or minimal scope of testing is certainly one of the sources of problems in this area.

At the same time, it is easy to understand why this is the case. The gradual increase in the number of computers in primary and secondary schools creates a large and tempting market. The schools, teachers, students, and their parents represent strong marketing groups, and furthermore the bills are conveniently paid for by the government. Thus it is not surprising that the major players in this field bring forth strong marketing tools, which often drown out and hide honest studies and any potentially unfavourable results.

On the other hand there is the objectively dire state of mathematical education in Slovakia. Taking a broad overview, the quality is unsatisfactory and mathematics is losing both its position and popularity. One of the key reasons is the offset between the breadth of the expected learning outcomes and the actual capabilities of the students, teachers, and schools. (A reduction of this breadth as a part of the current reform is from this point of view a reasonable measure.) Another reason is the wrong direction taken in the preparation of teachers, who are saddled with an overabundance of abstract mathematical knowledge and provided with desperately few real pedagogic tools for handling actual instruction. The consequence is many disappointed teachers, disinterested students, and unhappy parents and wider society.

In this situation, we have a tendency to believe and hope in a "magical" solution, something that ICT technologies currently may seem to offer. If we believe that the main strategy should be to make education more "attractive", then this route seems very promising. However, real solutions lie elsewhere, though quality e-learning can definitely contribute there as well. We must honestly explore both its opportunities as well as limitations, using proper scientific scepticism. That happens to be in conflict with the excitement and expectations stemming from the discovery of a potential happy solution to an unfortunate situation.

The discovery of a universal solution to all educational problems is a well known theme in mathematics, and one that should be treated with caution. We speak of a worldwide experience from the 1970's, that even thorough testing, comparing, and evaluation of methods of instruction, as well as enthusiasm and agreement of the testing instructors do not guarantee correct conclusions. During this time, under the influence of the success of structuralism, there was a decision to base mathematical instruction from the earliest grades on the concepts of set theory. From a strictly mathematical perspective, these present a unified foundation and starting point. The experiment took place in many countries: from France, to the USA, all the way to the countries of the Eastern Bloc. It was preceded by a pilot program with excellent results. Only in hindsight was it apparent that the enthusiasm of the teachers taking part in the experiment, their desire for new, modern teaching methods utilizing their own creativity, resulted in the results being in favour of the new teaching concepts. The subsequent large-scale implementation in schools was a fiasco, and after a few years the entire reform failed. More so than the testing, the objections raised by a minority, questioning the didactic suitability of the methods almost from the very start, proved to be accurate.

V. DOES E-LEARNING SAVE US WORK AND EXPENSES?

The possibility of saving the work, effort, and energy expended during teaching is often a big temptation of new technologies. If it however becomes their main goal, we invite mistakes and errors.

We would certainly be surprised should physical education be taught in schools by having the students watch a video of a somersault, then having them correct the somersaulting mistakes of an interactive avatar using a computer, and finally writing an exam about the correct way to perform a somersault. (This method would be correct if we were learning *about* physical education, or if we were trying to lead an active gymnast towards a more perfect motion. In the general population, it would however lead to a high risk of injury when attempting an actual somersault.) This would likely be considered nonsense by everyone, as sporting skills must be acquired through individual activity.

There is no reason to believe that mathematics is any less challenging than physical education, and the thought processes of the students can be replaced by a computer. Nevertheless, very seriously intended attempts at precisely this are common today. From experience and the results of PISA testing, we know that students have difficulties with word problems, especially with understanding a more extensive, context-based written problem, and its transformation all the way into the correct calculation leading to obtaining the solution. Our students need to improve and develop this ability. What should we then think of an e-learning product that displays a word problem, reads it, then announces the proper steps for solving it, and finally visually demonstrates them and verbally comments. The procedure furthermore utilizes multimedia tools, so the student cannot replicate it on paper. Instead of commenting, I would like to remind you of the somersault.

The temptation to unburden the student of work, to accelerate the course of instruction and make it more pleasant, is very strong. However, it is very important to carefully consider whether we are truly unburdening the student of activity that unnecessarily hinders him or her from progress towards an important goal. An example can be various computer-aided experiments (statistical, probabilistic, numeric, geometric), where keeping track of parameters and results can be simply a work-intensive burden. We must not however make a mistake and entrust to a computer any activity of a student that is a fundamental part of the learning process, whether it is the practicing of certain routine operations, or conversely the attempt to discover and formulate important properties of prime numbers or polygon diagonals.

What is the situation when it comes to saving work on the teacher's side of things? As an extreme example we can consider opinions such as "*If I record a video of my own lesson, I can then play it back for students in future years and don't even have to show up!*" These kinds of ideas are not only a degradation of e-learning, but of education in general. On the other hand, we must admit that some teachers might obtain better results with students should they replace themselves with a video. In our own experiences with utilizing e-learning, the majority of cases brought us more work, more effort, and more energy required while teaching. The first step is of course the necessity to acquaint oneself with the utilized tools, and then monitoring and managing their updates. We have already mentioned the heightened demands on the teacher's preparation, but this is also the case during the course of instruction. ICT technologies allow for much more convenient methods of submitting, grading, and checking of student assignments. Instead of several submitted assignments per semester, we now find ourselves checking dozens every week. The option to electronically publish study and work materials is convenient, however it leads to the need for their timely creation, continuous adjustments according to the current progress of instruction etc. We could continue with similar examples for quite some time.

At first glance from this point of view, the utilization of e-learning seems counterproductive. However, this passage is neither a lament nor a complaint. The extra work necessary has returned to us where it matters: in the form of smarter, more educated, and happier students. In summary, the main goal of e-learning tools should not be saving the teacher work, but instead to offer new tools for improving the learning process.

Besides considerations about saving work for students and teachers, there is another matter to consider: that of financial expenses. It is obvious that computers are more expensive than chalk and chalkboards, but usually we are assured that the costs are acceptable and proportional to the benefits. We discussed the measuring of said benefits in the preceding section. Here we would like to warn that it is necessary to be aware of all the expenses related to the operation of e-learning classrooms and equipment. The initial expenses are often covered by a project or program. Each device however represents a budgeting load that is hard to then get rid of. The majority of technology today is obsolete within a matter of just a few years and needs to be fully replaced. Another load is the cost of operation and repairs. That is why investing in equipment is a long-term financial commitment. Hardware innovation also brings with it the need for continual learning software innovation. Special products with a multimedia and interactive focus are very sensitive to changes in peripherals, input devices, operating systems etc.

These arguments relate not only to schools, but also the households of the students. This much is suggested through various ideas of replacing textbooks, workbooks, and homework with accessing learning programs and portals from home. Furthermore, if we do not establish a parent's responsibility to purchase a computer by law, we create an intriguing customer - the government - who can do it for everyone. In a post-secondary environment, these kinds of intentions have already appeared. Also taken into account must be the opinion of a parent from a discussion about a quality, functioning learning software: "*It's a nice program, but my son sits at his PC after school more than enough as it is. I don't want him to do this at school and for school as well, when there are lots of alternative learning tools and methods.*" Not every parent believes that just because something is done on a computer at school it is necessarily better, or better for their kids.

necessarily better, or better for their kids.

VI. Does E-Learning Help Mathematics?

Definitely yes, but it must be used in moderation, avoiding mistakes and staying aware of other considerations. ICT is irreplaceable mainly in making routine repetitive actions easier: transmitting organisational information to the students (due dates, instructions, announcements), ongoing and flexible publishing of study materials, directing the submission of homework, formalizing and de-personalizing exams (the teacher is an ally of the student, preparing him for the test, which can then thanks to technology be administered objectively), announcing test results, student discussion about problems, and distance education for those unable to attend in person.

In a truly bad educational situation, it can even be an improvement to show recorded math lectures. However, we feel that this is advancement in the wrong direction. If we save the starving by feeding them a diet of fast-food, we solve one problem only to soon replace it with others.

When teaching, it is important to take into account that we are working with different target groups. For some students mathematics will only be necessary for everyday life, for others it is an aid (sociology, medicine), a working tool (computer science, engineering, meteorology), and only for a select few: their profession and main area of interest (mathematics, physics). The approach for each of these differs, affecting the content and breadth of instruction. Of course, at the elementary school level we cannot yet determine which category a student will fall into. Therefore it is a problem with non-trivial solutions.

Well taught mathematics can also develop more universal, metacognitive aims. One of these is to create and cultivate the capability and methods of causal, logical thinking. Another, equally important, is to gain the experience of creative and original thinking, to experience the joy of an intellectual discovery. Our educational system should be required to provide these experiences for its students as a part of the schooling process. (Just as it should provide encounters with quality literature and music, foreign languages and culture, and the mastering and cultivating of certain physical activities.)

For these goals, mathematics provides sufficient, and compared to other subjects almost optimal, options and conditions.

Education based on the presentation of facts and subsequent training of standard solutions lead to mechanical repetition of learned procedures, the memorizing of formal knowledge, an inability to apply knowledge, connect and modify gained knowledge and abilities, and negative attitudes towards mathematics, which already manifest today in the decreased interest and quality of students interested in the natural sciences and technical disciplines.

VII. What Should the Creation of Useful Educational Software Look Like?

First of all, we should take advantage of the experience gained from already commenced educational reforms in countries where students' results have improved in international PISA testing. In the article [5], it is stated that the biggest improvement was seen among students from those countries where education was decentralized. For e-learning, this means that we shouldn't buy the same product for all schools, but instead give the teacher a choice. We can have a central repository of materials, which can then be selected from. In this database, teachers can access individual materials sorted based on the government educational aims that they can fulfill (sorted by difficulty, knowledge level, ability, and approach).

Optimal preparation of educational e-objects would consist of a teacher giving the requirements for a particular piece of software from his or her experience. A programmer would then create such an object. Of course a teacher may not always be aware of all that it is possible to program, or develop. The ideas of programmers can be interesting and the teacher may not be able to imagine all that it is possible to do. Therefore a mutual collaboration of teachers and developers should continue after the completion of the product. Modifications should be performed even after testing the product in classrooms.

Each software offered should also have a simple, reasonably priced version with intuitive controls. Expansions and new versions should be compatible with previous versions. Software for schools should be prepared by university students as a part of the completion of their studies and enthusiasts who understand education, not giant companies whose goal is to maximize profit. If we can ask teachers to teach for meager salaries, why can't we find developers also willing to consider their work a calling as opposed to simply a profitable endeavour.

It is becoming apparent that products created by teachers based on their experience together with programming enthusiasts are much more usable for teaching than professional products created with the goal of making money. In fact, these professional products often battle scholarly and didactic deficiencies, which are compensated for with investments into advertising and marketing. Students, and often even teachers are left defenseless against the lack of quality in e-learning. It is therefore necessary to support institutions that have the quality of education on their agenda, so that they make correct decisions in the direction and funding of the development of educational ICT technologies.

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